Collaborative Team Play

Scott Berger, Clear Lake High School
sberger@ccisd.net

New Twist on Math Circle Activities SIGMAA
MathFest, August 10, 2024

Goal is to have participants discuss, brainstorm, reason, persuade, and prove mathematical ideas in small groups.

Problem requirements:

- Easy-to-state –hit the ground running in a one-hour event
- Accessible – suitable for the range of math backgrounds and intensity levels in your audience
- Exploratory – everyone can make constructive or entertaining progress by puttering & brute force
- Open-middle – solvable through very distinct approaches, rather than bottlenecking on a specific ah-ha gimmick; all participants contribute rather than watching the leader.
- Instructive – contain insights into mathematically valuable concepts
- Fun – "the focus is on mathematical play"

FREE online Math Contests for middle-school and high-school students

Purple Comet! Math Meet website: purplecomet.org
Teams of up to six. Solve 30 problems in 90 minutes.

Rocket City Interschool Contest website: grissommath.org/rcml
Team of any size. Solve 15 problems in 30 minutes.

MathWorks Math Modeling Challenge website: m3challenge.siam.org
Teams of up to five; ages 16-19 only. Write a complete research paper in 14 hours.

USA Mathematical Talent Search website: usamts.org
Individual competition, but archived problems are good for team collaboration. Solve 5 problems including proof-writing in 1 month.

Texas A & M University Math Contest website: math.tamu.edu/outreach/highschoolcontest
Teams of up to six. Solve about 10 problems with proof-writing in 1 week. ($10 entry fee)

Online University of Houston Math Contest website: mathcontest.uh.edu
Individual competition (used to have team project, 2008-18). Was write project in 1 week.

American Regions Mathematics League website: arml3.com and arml.com/ARML/arml_2019/public_contest_files/2009_2014_book/ARML_2009_2014.pdf
Invitation-only teams of 15, but archived problems are good for collaboration. Multiple events including team, power team, and relay events in 6-60 minutes.

Mandelbrot Team Play (Purchasable problem book)
Teams of four. Solve about 5 problems with proofs in 1 hour.

Example Problems

(1) If you randomly break a meterstick into three pieces, what is the probability that the resulting pieces make a triangle?

(2) A camel transporting bananas can carry up to 1000 bananas on its back, but it must eat 1 banana per mile. If the merchant starts with 4000 bananas, what is the maximum number of bananas that can be transported across the 1000-mile-wide desert?

(3) Imagine that all the surfaces of a room are mirrored. Is a single light source always sufficient to illuminate the entire room? [Unilluminable rooms; billiard tables with curved cushions]

(4) A rectangular chocolate bar is marked into 4 x 10 squares by horizontal and vertical scores. Each turn, pick up one piece of the bar and break it along a score. What is the fewest number of turns needed to break entire chocolate bar into its constituent squares?

(5) A golden wedding ring is made by coring a cylindrical center out of a perfect sphere. When the resulting ring rests flat on the table, it has a height of 6mm. What is the total volume of the ring?

(6) A domino can be placed to cover any two adjacent squares on a chessboard. An 8x8 chessboard with left and right corner-squares removed can be covered by 31 dominoes. Show that 31 dominoes cannot be used to cover the chessboard missing its two diagonally-opposing squares. Which other boards with two squares removed cannot be tiled? [Domino tiling of mutilated chessboard]

(7) There are 100 students and 100 closed lockers. Student #1 goes down the row and opens every locker. Student #2 now closes every other locker (#2, 4, 6, etc.). Student #3 toggles every third locker: closing locker #3, opening locker #6, and so on. After all students have gone through, which lockers are open?

(8) Using arithmetic operations and the digits 2, 3, 4, 5 once each, generate all numbers from 0 to 100. [Four-fours, Krypto, and Twenty-Four]

(9) Chicken McNuggets come in 6-, 9-, and 20-nugget boxes. What is the largest number of nuggets that cannot be purchased by some combination? (For example, you can't buy 11 nuggets.) [Frobenius Coin problem, Sylver Coinage game]

(10) Given any positive integer n, prove that there is a positive integer multiple of n whose decimal representation contains only 0s and 1s. For example, 7 has the multiple: $1001 = 7 \times 143$.

(11) We want to determine the highest floor from which it is safe to drop an egg. We must design an algorithm to find the highest safe floor using only two test eggs. Testing one floor at a time from the ground up will do, but it's inefficient. What is the maximum number of times we must drop an egg (in the worst-case scenario)?

(12) Pick any whole number. If the number is even, divide it in half. If the number is odd, triple it and add one. Repeat forever. Do you always end up at 1?

Collaborative Team Play

Scott Berger, Clear Lake High School
E-mail: sberger@ccisd.net

New Twist on Math Circles

MAA MathFest Math Circle SIGMAA

August 10, 2024

Math puzzle problem examples

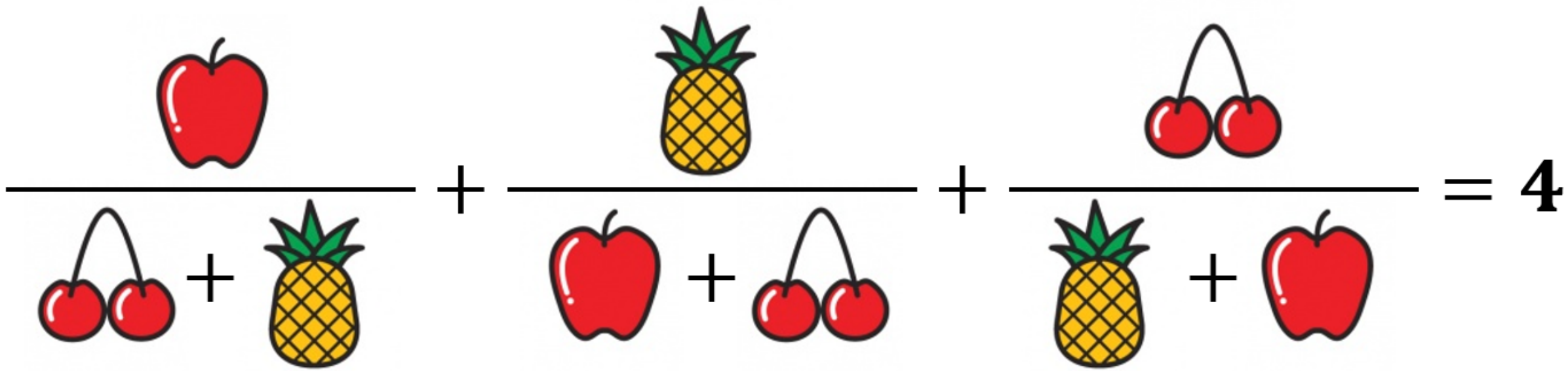
“If you randomly break a meterstick into three pieces, what is the probability that the resulting pieces make a triangle?”

“A camel transporting bananas can carry up to 1000 bananas on its back, but it must eat 1 banana per mile. If the merchant starts with 4000 bananas, what is the maximum number of bananas that can be transported across the 1000-mile-wide desert?”

“Imagine that all the surfaces of a room are mirrored. Is a single light source always sufficient to illuminate the entire room?”

“A rectangular chocolate bar is marked into 4 x 10 squares by horizontal and vertical scores. Each turn, pick up one piece of the bar and break it along a score. What is the fewest number of turns needed to break entire chocolate bar into its constituent squares?”

95% of people cannot solve this!



Can you find positive integer values for 🍎, 🍍, and 🍒?

Math circle activity: Students collaborate in groups to solve math puzzle problems

Goal is to have participants discuss, brainstorm, reason, persuade, and prove mathematical ideas in small groups.

Advance warning: I teach at a large public high school.

Key differences versus a college & community math circle:

(1) 8^{th} – 12^{th} graders rather than all ages. Also some A.I.M.E. qualifiers looking to be challenged, and

(2) Captive audience: if didn't finish or it didn't work, we can try again next week or next month

What makes a good math puzzle problem?

Requirements:

- Easy-to-state – hit the ground running in one-hour event
- Accessible, Low-Floor – suitable for range of math backgrounds and intensity levels
- Exploratory – all can make constructive progress by puttering & brute force
- Open-middle – solvable through distinct approaches; does not bottleneck on an “Ah-ha!” gimmick
- High-Ceiling – tremendous depth; dramatic unanswered questions
- Instructive – insights into mathematically valuable concepts
- Fun – “the focus is on mathematical play”

What resources do I need to host a Math circle?

Each column shows progressively more initial resources; you might be able to host a puzzle contest at column I or II (as opposed to III or IV)

Problem statement

Problem & Answer(s)

Framework:
Timeline of lesson activities

Printable physical manipulatives

Comprehensive:
Math ideas, vocabulary
Pedagogy, scaffolding
Extensions
History, open questions

Purchasable math circle activity kit

How much extra time and money do I have?

None.

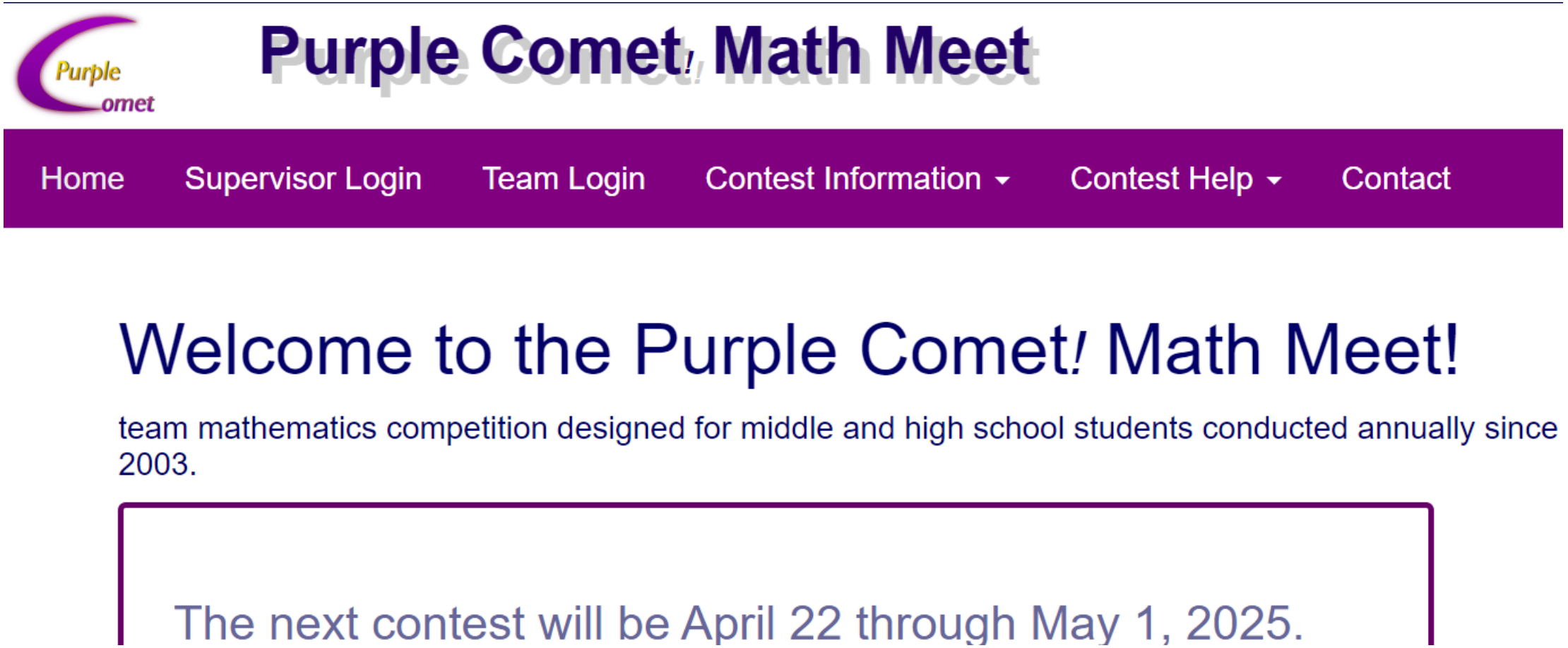
So I have a strong bias toward finding FREE materials.

Some useful tools (that I know about – tell me what I'm missing)

How many of you are familiar with [I mean familiar enough to use as a resource]:

- MAA Connect (connect.maa.org)
- Julia Robinson Mathematics Festival (jrmf.org)
- ChatGPT (or Claude or Gemini or Copilot or…)
- Desmos
- Desmos Activity Builder (like Marbleslides and Polygraph)
- Art of Problem Solving (artofproblemsolving.com)

A Fun Collaborative-Team Math Contest!



Purple Comet: Free online team contest

- Teams of 1 to 6
- 30 problems in 90 minutes
- Middle school and high school divisions (but all ages are welcome)
- Once per year in April
- Yes: Calculators and textbooks allowed
- Yes: Students can write programs
- No: No online communication, no internet searches, no LLMs

PURPLE COMET! MATH MEET April 2024
HIGH SCHOOL - PROBLEMS

Problem 1

Joe ate one half of a fifth of a pizza. Gale ate one third of a quarter of that pizza. The difference in the amounts that the two ate was $1/n$ of the pizza, where n is a positive integer. Find n.

Problem 2

Consider triangles whose three angles have three different positive integers for their degree measures. Find the greatest possible difference between the degree measures of two of the angles in such a triangle.

PURPLE COMET! MATH MEET April 2024
HIGH SCHOOL - PROBLEMS

Problem 8

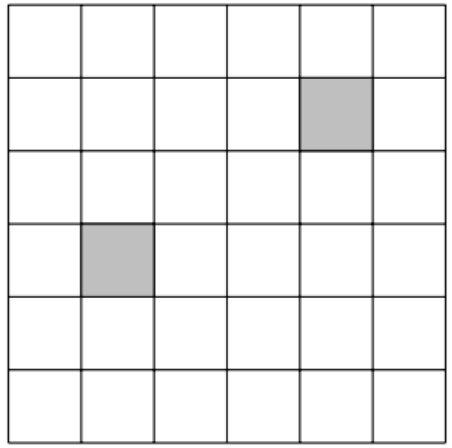
Let a and b be nonzero real numbers such that

$$(a-b)^3+(12a-b)^3=(9a-b)^3+(10a-b)^3.$$

The fraction a/b reduces to m/n, where m and n are relatively prime positive integers. Find $m+10n$.

Problem 9

Find the number of rectangles pictured in the rectangular grid that contain one but not both of the shaded squares.



FREE online Math Contests for middle-school and high-school students

Purple Comet! Math Meet website: purplecomet.org
Teams of up to six. Solve 30 problems in 90 minutes.

Rocket City Interschool Contest website: grissommath.org
Team of any size. Solve 15 problems in 30 minutes.

MathWorks Math Modeling Challenge website: m3challenge.siam.org
Teams of up to five; ages 16-19 only. Write a complete research paper in 14 hours.

USA Mathematical Talent Search website: usamts.org
Individual competition, but archived problems are good for team collaboration.
Solve 5 problems including proof-writing in 1 month.

Additional **FREE** online Math Contests for middle-school and high-school students

Texas A & M University Math Contest
website: math.tamu.edu/outreach/highschoolcontest
Teams of up to six. Solve 10 problems with proof-writing in 1 week. ($10 entry fee)

Online University of Houston Math Contest
website: mathcontest.uh.edu
Individual competition (used to have team, 2008-18). Was write project in 1 week.

American Regions Mathematics League website: arml3.com and arml.com/ARML/arml_2019/public_contest_files/2009_2014_book/ARML_2009_2014.pdf
Invitation-only teams of 15, but archived problems are good for collaboration.
Multiple events including team, power team, and relay events in 6-60 minutes.

Mandelbrot Team Play (Purchasable problem book)
Teams of four. Solve about 5 problems with proofs in 1 hour.

Example Math Puzzle Problems

These are some of my favorites; e-mail sberger@ccisd.net if you would like help finding resources or have resources to contribute!

(1) If you randomly break a meterstick into three pieces, what is the probability that the resulting pieces make a triangle?

(2) A camel transporting bananas can carry up to 1000 bananas on its back, but it must eat 1 banana per mile. If the merchant starts with 4000 bananas, what is the maximum number of bananas that can be transported across the 1000-mile-wide desert?

(3) Imagine that all the surfaces of a room are mirrored. Is a single light source always sufficient to illuminate the entire room? [Unilluminable rooms; billiard tables with curved cushions]

Example Math Puzzle Problems

(4) A rectangular chocolate bar is marked into 4 x 10 squares by horizontal and vertical scores. Each turn, pick up one piece of the bar and break it along a score. What is the fewest number of turns needed to break entire chocolate bar into its constituent squares?

(5) A golden wedding ring is made by coring a cylindrical center out of a perfect sphere. When the resulting ring rests flat on the table, it has a height of 6mm. What is the total volume of the ring?

(6) A domino can be placed to cover any two adjacent squares on a chessboard. An 8x8 chessboard with left and right corner-squares removed can be covered by 31 dominoes. Show that 31 dominoes cannot be used to cover the chessboard missing its two diagonally-opposing squares. Which other boards with two squares removed cannot be tiled? [Domino tiling of mutilated chessboard]

Example Math Puzzle Problems

(7) There are 100 students and 100 closed lockers. Student #1 goes down the row and opens every locker. Student #2 now closes every other locker (#2, 4, 6, etc.). Student #3 toggles every third locker: closing locker #3, opening locker #6, and so on. After all students have gone through, which lockers are open?

(8) Using arithmetic operations and the digits 2, 3, 4, 5 once each, generate all numbers from 0 to 100. [Four-fours, Krypto, and Twenty-Four]

(9) Chicken McNuggets come in 6-, 9-, and 20-nugget boxes. What is the largest number of nuggets that cannot be purchased by some combination? (For example, you can't buy 11 nuggets.) [Frobenius Coin problem, Sylver Coinage game]

Example Math Puzzle Problems

(10) Given any positive integer n, prove that there is a positive integer multiple of n whose decimal representation contains only 0s and 1s. For example, 7 has the multiple: $1001 = 7 \times 143$.

(11) We want to determine the highest floor from which it is safe to drop an egg. We must design an algorithm to find the highest safe floor using only two test eggs. Testing one floor at a time from the ground up will do, but it's inefficient. What is the maximum number of times we must drop an egg (in the worst-case scenario)?

(12) Pick any whole number. If the number is even, divide it in half. If the number is odd, triple it and add one. Repeat forever. Do you always end up at 1?

Thank you for your interest!

Collaborative Team Play

Scott Berger, Clear Lake High School

E-mail: sberger@ccisd.net

New Twist on Math Circles

MAA MathFest Math Circle SIGMAA

August 10, 2024